Lesson 6.3 – Proving a Quadrilateral is a Parallelogram

Organizing all this stuff-keep at it!

Do it again. I know you don't want to, but do it. Just do it.

Take out a piece of paper. Lay it down long ways (landscape style). On this sheet, draw the entire quadrilateral family tree from memory. Write out the properties of parallelograms we've learned. There should be eight basic properties, three of which are just the converse of others. So really, all you need to do is remember four and be able to come up with the converses.

Converse … it isn't just a tennis shoe

Write out theorem 6.1 as best as you can from memory. State it as a conditional. It is the one dealing with opposite sides. Once you are done, check it against the book. Now write the converse. This is theorem 6.7 … check it against the book.

Do the same with theorem 6.2, the one dealing with opposite angles. The converse is theorem 6.8. Check against the book and make any corrections necessary.

Now, one more time with theorem 6.3; it's the one dealing with diagonals. Write the converse which turns out to be theorem 6.5.

Keeping it straight

We now have two sets of theorem for parallelograms:

- One set tells us what a parallelogram is like…if you are told something is a parallelogram, you now know what you can expect of it.

- The other set tells us that if we see these characteristics (opposite sides congruent, etc.) then we can determine that the shape is a parallelogram. This is the converse set.

The converse set is used to prove some quadrilateral is (or isn't) a parallelogram. The "regular" set is used to draw conclusions about a ***known*** parallelogram.

Making our lives a bit easier…

If we want to prove a quadrilateral is a parallelogram, we need both pairs of opposite sides (or angles) congruent. Can we get away with less? What if we only knew that one set of opposite sides was congruent. Would that be enough?

If you play around with it drawing a few pictures, you will quickly realize it isn't. Draw an isosceles trapezoid. It has one set of congruent opposite sides doesn't it? Hmm, we found a counter example which proves the conjecture wrong.

So we need more info. What would help us out here? What if the congruent sides were also parallel? Spend a minute or two and see if you can come up with a counter example.

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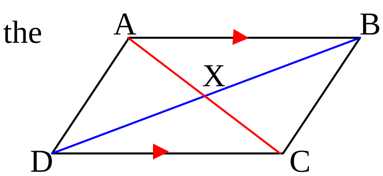
Hmm, there is no counter example. Let's see if we can use deductive reasoning to prove this conjecture…let's do a proof.

Given: Quadrilateral $ABCD, \overline{AB} \cong \overline{CD}, \overline{AB} \parallel \overline{CD}$

Prove: Quadrilateral $ABCD$ is a parallelogram.

Proof: Well first off, what tools do we have at our disposal? We have a quadrilateral we want to prove is a parallelogram. That means we need to look at the converse set of theorems (6.5, 6.7 and 6.8). For instance, if we can prove the diagonals of this quadrilateral bisect each other, then we can assert it is a parallelogram.

Let's start off by constructing the diagonals $\overline{AC}$ & $\overline{BD}$ and call the point at which they intersect point X. Here is a diagram.



If we can show $\overline{AX} \cong \overline{XC}, \& \overline{DX} \cong \overline{XB}$ then we'll have shown the diagonals bisect each other. To do that, let's prove $\Delta AXD \cong \Delta CBX$ and use CPCTC.

$\overline{AB} \parallel \overline{CD}$	given	
$\overline{AC}$ & $\overline{BD}$	are transversals of parallel lines	
$\angle DAX \cong \angle BCX$	alt int $\angle$'s $\cong$	(***angle***)
$\overline{AB} \cong \overline{CD}$	given	(***side***)
$\angle ADX \cong \angle CBX$	alt int $\angle$'s $\cong$	(***angle***)
$\therefore \Delta AXD \cong \Delta CBX$	ASA	
$\overline{AX} \cong \overline{XC}, \& \overline{DX} \cong \overline{XB}$	CPCTC	
Q.E.D.		

We have proven a new theorem…let's add it to our tool box as theorem 6.6.